

Supersymmetric Large Extra Dimensions and the Cosmological Constant: An Update

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ABSTRACT: This article critically reviews the proposal for addressing the cosmological constant problem within the framework of supersymmetric large extra dimensions (SLED), as recently proposed in [hep-th/0304256](#). After a brief restatement of the cosmological constant problem, a short summary of the proposed mechanism is given. The emphasis is on the perspective of the low-energy effective theory in order to see how it addresses the problem of why low-energy particles like the electron do not contribute too large a vacuum energy. This is followed by a discussion of the main objections, which are grouped into the following five topics: (1) Weinberg's No-Go Theorem. (2) Are hidden tunings of the theory required, and are these stable under renormalization? (3) Why should the mechanism apply only now and not rule out possible earlier epochs of inflationary dynamics? (4) How big are quantum effects, and which are the most dangerous? and (5) Even if successful, can the mechanism be consistent with cosmological or current observational constraints? It is argued that there are plausible reasons why the mechanism can thread the potential objections, but that a definitive proof that it does depends on addressing well-defined technical points. These points include identifying what fixes the size of the extra dimensions, checking how topological obstructions renormalize and performing specific calculations of quantum corrections. More detailed studies of these issues, which are well reach within our present understanding of extra-dimensional theories, are currently underway. As such, the jury remains out concerning the proposal, although the prospects for acquittal still seem good. (An abridged version of this article appears in the proceedings of *SUSY 2003*.)

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1. What is So Hard About the Cosmological Constant Problem?

The current evidence for a small but nonzero cosmological constant [1] underlines the complete absence of a convincing theoretical understanding for why this constant of nature can be so small. On the one hand, observations now indicate the vacuum energy density is of order $\rho = v^4$, with $v \sim 3 \times 10^{-12}$ GeV in units for which $\hbar = c = 1$. On the other hand all known theories of microscopic physics appear to predict that a particle of mass m contributes to ρ an amount which is of order $\delta\rho \sim m^4$. The problem arises because almost all currently known elementary particles have masses much larger than 10^{-12} GeV. (See [2] for a review of past heroic attempts to solve this problem, together with a no-go theorem.)

As is the case for any ‘naturalness problem’, any fundamental solution to the cosmological constant problem must answer two questions.

1. Why is the vacuum energy, ρ_0 , so small at the microscopic scales, $M \gtrsim 10^3$ GeV, at which the fundamental theory is couched?
2. Why does it remain small as all the scales between M and v are integrated out?

It is the second of these problems which is the more worrying, because it seems to indicate that we are missing something in our description of *low energy* physics, which we normally think we understand. For instance, we believe we understand the electron quite well and yet even in this supposedly well-understood theory we do not know why the electron doesn't contribute the catastrophically large amount $\delta\rho \sim m_e^4$, with $m_e = 5 \times 10^{-4}$ GeV. At present our only theoretical refuge is that of the desperate: we claim that the microscopic theory predicts an enormous cosmological constant, $\rho_0 \sim M^4$, and that this gets systematically cancelled (to at least 60 decimal places!) by the process of integrating out all of the lighter particles down to the very low energies where ρ is measured.

Supersymmetric theories provide the only partial ray of light in this otherwise totally dark picture. This is because sufficiently many supersymmetries can explain why the microscopic value, ρ_0 , must be small or vanish (thereby addressing problem 1). It can also partially explain why the process of integrating out lighter particles does not ruin this prediction (problem 2), because supersymmetry enforces a cancellation between bosons and fermions in their contributions to ρ . Unfortunately, this cancellation is only partial if supersymmetry is broken, leaving a residual value which can be as small as $\delta\rho \sim m_{sb}^4$, where m_{sb} is a measure of the largest mass splittings between bosons and fermions within a supermultiplet. Sadly, experiment already implies that this scale must be at least as large as $m_{sb} \sim 10^2$ GeV for observed particles like electrons.

2. Supersymmetric Large Extra Dimensions (SLED)

Problems like the cosmological constant problem, which resist solution for many years, normally do so because they involve a hidden assumption which everyone has made, and which unnecessarily restricts the kinds of theories which could be considered. If only such an assumption could be identified, one could ask whether its relaxation might help reduce the size of the predicted cosmological constant.

Crucially, any such assumption must already be playing a role in the description of physics above the scale $v \sim 10^{-12}$ GeV, set by the cosmological constant itself. At present this requirement is a hindrance because physics at these scales is heavily constrained by observations, making the search for the successful theory more difficult. Ultimately it will be a boon, however, since any successful proposal must change physics at these low energies in many ways, and so must inevitably make testable low-energy predictions in addition to explaining why ρ is so small. This is certainly true of the scenario which is of interest in this paper.

2.1 The Assumption of 4 Dimensions

In ref. [3] it is proposed that the hidden assumption which might be behind the cosmological constant problem is the assumption that physics is four dimensional

at energy scales above v . After all, this assumption is crucial to the problematic statement that a particle of mass m contributes an amount $\delta\rho \sim m^4$. If the world were to involve extra dimensions as large as $r \sim 1/v \sim 0.1$ mm, then any calculation of the contribution of a particle having mass $m > v$ must be done in a higher-dimensional theory for which the predictions may differ. This supersymmetric large-extra-dimensional (SLED) picture is based on the earlier non-supersymmetric large-extra-dimensions (LED) proposal [4], the authors of which first realized that the world could be extra-dimensional down to such extremely low energies.

Extra dimensions can be as large as $r \sim 1/v$ without being in conflict with experimental observations provided that one works within the string-motivated brane-world framework, where all observed interactions except gravity are trapped on a 4-dimensional surface (brane) within a 6-dimensional spacetime [4]. Since in this picture only gravitational interactions can probe the existence of the extra two dimensions, these dimensions can be much larger than had been hitherto believed. It happens that the current limit on the size of these two dimensions, based on the best present measurements of the gravitational inverse-square law [5], is about $r \sim 1/v \sim 0.1$ mm.

The SLED picture can work, but only if there are precisely two extra dimensions which are as large as $1/v$, and only if the scale of gravitational physics in the extra dimensions is as low as is possible: $M \sim 10^3$ GeV. This last requirement is required because Newton's constant, $8\pi G = 1/M_p^2$, is related to r by $M_p \sim M^2 r$, and so $M = (M_p/r)^{1/2} \sim 10^3$ GeV.¹ Since M is near the weak scale, it follows that any SLED proposal is likely to have an interesting phenomenological signature in collider experiments at the LHC [6, 7] just as is true for the LED picture [8, 9, 10].

Both the LED and SLED scenarios are also subject to astrophysical bounds [8, 12, 13, 6], but we temporarily put these aside to see whether progress can be made with the cosmological constant. If so, we are confident that model-dependence of the astrophysical arguments can be exploited to evade the astrophysical bounds.²

2.2 Classical Contributions to ρ

Can extra dimensions help us understand why the electron does not contribute too large a vacuum energy density? It turns out that they can. Since observations require that supersymmetry is badly broken on the brane, within the SLED framework integrating out the electron (or any other observed particles) indeed does contribute a vacuum energy density which is of order m^4 . But this energy density is *not* a cosmological constant. Rather, it is a contribution to the tension of the brane: $\delta T \sim m^4$. That is, it is an energy source which is localized within the extra dimensions at the position of the brane on which we live. We must ask how this energy source

¹We use here the Jordan frame, for which M_p is r dependent but the electroweak scale, M , is not. It is in this frame that the Kaluza-Klein mass scale is $m_{\text{KK}} \sim 1/r$.

²See, for instance, ref. [14] for a sample way around some of these constraints.

curves the extra dimensions, and then see what the implications are for the effective 4D cosmological constant which would be observed on distance scales larger than $r \sim 0.1$ mm.

It happens that the curvature of the extra dimensions due to this localized energy source can be computed, with the result that the geometry acquires a conical singularity at the position of the branes, with a correspondingly singular contribution to the two-dimensional curvature given by³

$$R_2 = -\frac{2}{e_2} \sum_i T_i \delta^2(y - y_i) + \dots \quad (2.1)$$

Here $e_2 = \sqrt{\det g_{mn}}$ is the volume element for the internal two dimensions, y_i denotes the position of the ‘ i ’th brane and the ellipses denote contributions to R_2 which are smooth at the position of the brane.

How do the tension and curvature contribute to the effective 4D cosmological constant on long distance scales? This is obtained by integrating out the bulk gravitational degrees of freedom, which are not localized on the branes. If these are integrated out at the classical level an interesting cancellation occurs between the brane tensions and the bulk curvature. The effective 4D cosmological constant obtained at this order is

$$\begin{aligned} \rho_{\text{cl}} &= \sum_i T_i + \int_M d^2y e_2 \left[\frac{1}{2} R_2 + \dots \right] \\ &= 0, \end{aligned} \quad (2.2)$$

where the sum on ‘ i ’ is over the various branes in the two extra dimensions and ‘ \dots ’ denotes all of the other terms besides the Einstein-Hilbert term in the supersymmetric bulk action. Interestingly the sum over brane tensions, T_i , precisely cancels the contribution of the singular part of the curvature, eq. (2.1), to which they give rise [16]. Remarkably, the same kind of cancellation also occurs amongst the remaining terms in ρ_{cl} once these are evaluated using the smooth parts of the geometry and the other bulk fields obtained using the classical field equations [3].

We see there is a cancellation in ρ between the contribution of the brane tensions and the extra-dimensional curvatures to which these tensions give rise. Better yet, this cancellation does not depend at all on the *value* of the brane tension, and so applies equally well even if the T_i are large and include all of the quantum effects due to virtual particles localized on the branes.

2.3 Bulk Loops

Given that classical contributions cancel, what of the quantum effects? Since the quantum effects on the brane can be regarded as being included in the value of the

³This and later expressions use Weinberg’s curvature conventions [15]

effective brane tension, the quantum effects which remain are those involving the particles which live in the bulk. These include the graviton and all of its partners under higher-dimensional supersymmetry.

These bulk particles have several properties which are crucial for understanding the size of their contributions to the effective 4D cosmological constant, ρ , and which follow from the fact that they are related by supersymmetry to the graviton. The most important such property is the scale of supersymmetry breaking in the bulk, since this governs the effectiveness of the quantum cancellations in the effective 4D cosmological constant. Remarkably, this scale is naturally of order $m_{sb} \sim v$ as may be seen by either of two arguments. First, this scale follows because supersymmetry breaks on the brane at scale M , to which each bulk mode only couples with (4D) gravitational strength (because they are related to the graviton by supersymmetry). Consequently $m_{sb} \sim M^2/M_p \sim v$. Alternatively it follows from an explicit Kaluza-Klein reduction, because there is one massless graviton mode but the presence of the branes forces the lightest gravitino mode to be of order the Kaluza-Klein scale, $m_{sb} \sim 1/r \sim v$. The agreement of these two lines of reasoning relies on the value of r required by the hierarchy problem, inasmuch as this ensures $M^2/M_p \sim 1/r$.

The SLED proposal for understanding the small cosmological constant is founded on the assertion that the quantum contributions of bulk fields to the vacuum energy are of order $\delta\rho \sim m_{sb}^4$. In this case we see that bulk loops would predict a cosmological constant whose magnitude agrees with observations. Although m_{sb}^4 is arguably a reasonably generic estimate for a vacuum energy in a supersymmetric theory, a key part of the argument is a detailed justification of this estimate. This is particularly so, given that the supersymmetry-breaking fields are extra-dimensional particles which do couple nontrivially to the supersymmetry-breaking physics on the branes. A closer examination of this point is one of the discussion items in the next section.

There is indirect evidence that the simple estimate $\rho \sim m_{sb}^4$ is correct. This evidence comes from direct one-loop string-based calculations of the vacuum energy in toroidal compactifications with supersymmetry broken by boundary conditions and/or the presence of branes [17]. Such explicit calculations indeed find a vacuum energy which is of order $\rho \sim m_{sb}^4 \sim 1/r^4$. These calculations leave several issues open, however, such as whether the result is an artefact of the one-loop approximation or of the use of toroidal geometry. It would be preferable to be able to complement them with more general arguments based on power-counting within the low-energy theory, and so we return to this issue in the next section.

3. A Closer Inspection

Although the picture presented in the previous section is appealing, it must address several issues in order to be accepted as a real solution to the cosmological constant

problem. These issues are summarized in this section, together with a discussion of the likelihood that they are likely to be addressed by the SLED proposal.

3.1 Scale Invariance and Weinberg’s No Go Theorem

The most interesting feature about the classical vanishing of ρ described above is that the cancellation between brane and bulk contributions occurs for *any* value of brane tension. This is reminiscent of a similar cancellation which also has been found to occur in some 5-dimensional models [18], and one wonders whether they occur for similar reasons. It turns out they are indeed similar, because in both cases the cancellation of the effective 4D cosmological constant can be traced to the existence of an underlying classical scale invariance of the bulk gravity or supergravity action. Provided this symmetry is not broken by the brane couplings, the classical contribution to ρ in 4D is guaranteed to vanish [19]. (More precisely, for brane actions which — in the 6D Einstein frame — take the form

$$S_b = -T_3 \int d^4x e^{-\lambda\phi} \sqrt{-g}, \quad (3.1)$$

where ϕ is the dilaton which appears in the 6D gravity supermultiplet, the condition that the brane couplings do not break the classical scale invariance of the bulk theory requires $\lambda = 0$.)

Because the cancellation leading to vanishing ρ in the classical integration over bulk fields relies on a classical scale invariance, the cancellation need not survive the quantum part of the bulk integration. The difference between SLED and the 5D models is that for SLED there is a bulk supersymmetry which can plausibly protect the 4D cosmological constant from receiving corrections which are too large, since the supersymmetry breaking scale in the bulk is precisely $m_{sb} \sim v$. There is no such symmetry at work in the 5D models (or models in other dimensions), and so for these theories there is no reason to expect that the self-tuning should survive quantum effects in the bulk.

The reliance on classical scale invariance also brings to mind Weinberg’s No-Go theorem. In ref. [2] Weinberg raises a very general objection against so-called self-tuning solutions to the cosmological constant problem, which in their essence rely on scale invariance of the underlying equations. In cartoon form, his objection proceeds as follows. Even if scale invariance is a good symmetry at the quantum level, it must be spontaneously broken because physical scales like particle masses are known to exist. There must therefore be a goldstone (or pseudo-goldstone) boson for scale invariance, σ ,⁴ which transforms under scale transformations like $\sigma \rightarrow \sigma + \epsilon$. But although this transformation can forbid a cosmological constant, it does not solve the cosmological constant problem since it cannot prevent the generation of a scalar

⁴Confusingly, σ is called a dilaton, but should not be confused with the 6D scalar field, ϕ , which is related to the metric by supersymmetry in six dimensions.

potential of the form $V_{\text{dil}} \propto e^{c\sigma}$, for some constant c . However if such a potential exists, then σ gets driven to infinitely large values, which corresponds to the scale-invariant vacuum (*i.e.* the one which does not spontaneously break scale invariance). Although such a theory does have a vanishing cosmological constant, it does so at the cost of having all masses vanish because of the unbroken scale symmetry. This is not a successful theory of the world which is revealed to us by experiments.

Because the argument is explicitly couched in 4 dimensions, it cannot describe the generation of a localized brane tension by integrating out brane particles (like the electron above). It can and does describe the physics of the cancellation between the resulting localized tension on the branes and the curvature which these tensions set up in the extra dimensions. It does so because this classical cancellation is based on a classical scale invariance [19]. Weinberg’s argument then correctly says that this invariance cannot save us from quantum corrections which generate a potential energy for the dilaton, σ . This is true, and in the SLED case implies that the quantum corrections are likely to generate a potential for the classical flat direction which is parameterized by the particular combination of the radius and 6D dilaton field, $s = e^\sigma = e^\phi/r^2$ [20, 21]. Understanding this potential is a crucial step towards understanding what dynamics stabilizes the size of the extra dimensions to $r \sim 1/v$. Because it is supersymmetry – and not scale invariance – which in the SLED scenario enforces the bulk bose-fermi cancellations which keep ρ small, this part of the mechanism is not affected by Weinberg’s argument. Furthermore, because the scale invariance is broken by quantum effects, the potential which bulk loops produce need not have the simple form $e^{c\sigma}$, and so need not necessarily imply a runaway to a scale invariant vacuum. (Whether it actually does predict such a runaway is a more detailed question, more about which in later sections.)

3.2 Hidden Fine Tunings?

An advantage of the 6D SLED proposal is the ability to solve the back-reaction problem and write down explicit solutions to the bulk equations of motion, including the gravitational effects of the branes. As such it is possible to more closely examine the nature of the solutions, and so to see if the classically vanishing vacuum energy has somehow been ensured through a hidden fine-tuning of some of the parameters of the theory. This kind of hidden fine-tuning has been raised as an objection [22] to the 5D models of ref. [18].

Indeed, at first sight there does seem to be such a tuning going on in the SLED solutions [3, 23]. It is the purpose of this section to describe this tuning, and to argue why it need not be a problem for the SLED proposal for the cosmological constant problem.

For these purposes a concrete example is helpful, and the simplest one to think about in this regard is the ‘rugby ball’ solution of the gravity [24] or supergravity [3] equations. Here the geometry of the internal dimensions is a 2-sphere threaded by

the flux of a magnetic monopole, which generalizes an old compactification [25, 26] of 6D Nishino-Sezgin supergravity [27]. The solution includes two branes, which can be located at the sphere's north and south pole. Their back-reaction onto the geometry is then described by removing a wedge from the two sphere and identifying opposite sides of the wedge. Einstein's equations imply that the angular width of the defect at each end of the wedge must be strictly proportional to the tension of the brane at that end. But such a wedge can only be removed from a sphere without curving it if the boundaries of the wedge are lines of fixed longitude on the sphere, and so the defect angle at both ends of the wedge must in this case be equal. The spherical solution therefore requires the tension of the two branes to be positive and to be precisely equal.

The bulk geometry can also be found for 6D supergravity even if the brane tensions are not equal [28]. It is found in this case that both the warping of the 4D metric and the 6D dilaton field acquire a nontrivial variation across the extra dimensions. There is a generalization of the equal-tension constraint also for this geometry, however, which relates the tensions of the two branes to various bulk quantities. This constraint simply expresses the topological statement that the Euler number of the internal geometry is the same as for a 2-sphere [28, 19].

A similar constraint may also be derived involving the gauge coupling constants and the amount of magnetic flux threading the sphere [3], and this second constraint also has a topological interpretation. It expresses the statement that the Chern class of the 2-dimensional magnetic field is the same as for a magnetic monopole.

Relations such as these between the tensions of the branes are also reminiscent of the situation in 5 dimensions. There, solutions are found to the bulk Einstein equations in the presence of a single brane, for which the effective 4D geometry is also flat regardless of the value of the brane tension. However in this case the bulk solutions are found necessarily to be singular, and this singularity can be interpreted as the response of the geometry to a second brane whose existence is required by the solution. Since it turns out that the tension of this second brane is equal and opposite to the tension of the first brane, their tensions cancel in the 4D vacuum energy and the flatness of the 4D space is explained by an apparent self-tuning. This 5D constraint of opposite tensions for the branes is also topological in nature.

Does the existence of such constraints kill the SLED proposal for solving the cosmological constant problem? Not necessarily, for the following reasons.⁵

The central point to keep in mind here is version 2 of the cosmological constant problem as given above. One issue which this raises is whether the effective 4D cosmological constant remains zero as successive scales are integrated out from a high scale, say $M \sim 10^3$ GeV to the low scale $v \sim 10^{-12}$ GeV. We must ask: If a constraint among the tensions or magnetic fluxes is imposed in the short distance

⁵There is a *caveat* to the argument given here, concerning the warping of the bulk geometry, which is discussed in a later section.

theory at distances of order $1/M$, does it remain imposed as we renormalize down to long distances of order $1/v$? If so, then an understanding of the smallness of the cosmological constant can be consistently deferred until the theory describing physics at energy M is understood.

Although this renormalization has not yet been performed explicitly for these models, there is a simple line of reasoning which argues that the constraints we are considering (like equality of the tensions) should be stable against integrating out the scales between M and $1/r$. The key point is that all of these constraints are topological in origin, in the 6D case being related to the expression for the Euler number or the Chern class of the internal field configurations. They take the generic form $F(T_1, T_2, r, \dots) = n$, where the left-hand-side is a function, F , of the brane tensions and charges as well as of various bulk quantities. Because the right-hand-side is an integer, the quantity F cannot renormalize even if its arguments do. As such, these constraints cannot be changed by the integrating out of modes whose wavelengths are very short compared with the size r of the internal space. Integrating out very short wavelength modes can renormalize *local* operators in the 6 dimensions, but do not ‘know’ about the topology because their wavelengths do not reach completely across the internal space.⁶

There is an indirect check that the vanishing of the classical vacuum energy is stable to perturbations to special features of specific solutions (like equality of tensions on the two branes). The check comes from the discovery of a very general class of axially-symmetric solution for the two-brane configuration in 6D supergravity [28]. These describe the back-reaction which is appropriate to an arbitrary pair of brane tensions (but excludes the types of direct dilaton and magnetic couplings to the branes discussed in ref. [3]), and so allow one to ask whether the classical tension-cancellation mechanism remains satisfied in a wider context. Most remarkably, despite their not being supersymmetric, for *all* of these solutions the 4 dimensions seen by brane observers are found to be precisely flat, regardless of the relative size of the tensions on the two branes, in agreement with the general scale invariance arguments of ref. [19].

3.3 The ‘Moving Target’ Problem

Is the SLED mechanism too much of a good thing? That is, does it act to zero the effective cosmological constant at all times during cosmology or just at present? If it were to zero the cosmological constant at all times it would be difficult to understand how inflation could ever arise in the universe’s history – what is being called here the ‘moving target’ problem.

That this is not a worry follows from the observation that the radius, r , of the extra dimensions is unlikely to always have been as large as it is now. Although r

⁶It is this property which underlies the general expression of ultraviolet effects in terms of a local curvature expansion [29].

cannot have changed much since the epoch of Big Bang Nucleosynthesis (more about this later), it is very likely to have evolved considerably — probably growing from much smaller values — at much earlier times. If so then the prediction $\rho \sim 1/r^4$ need not imply that ρ must be negligibly small at epochs of cosmology earlier than nucleosynthesis.

A related question asks what happens if the tensions on the two branes in the rugby ball solution suddenly change, as they would once one considers them to be field-dependent quantities associated with the scalar potential for various fields localized on the branes. In particular, there could be vacuum phase transitions taking place on the branes, such as the electroweak or QCD phase transition, or an earlier transition possibly associated with baryogenesis or other higher-energy physics. Since the tensions on the two branes are unlikely to evolve identically, how can the various topological constraints continue to be satisfied?

It is possible to answer this issue in some detail at the classical level, because of the discovery of the more general geometries containing a pair of branes having different tensions [28, 19]. If the brane tensions were to spontaneously change then the bulk geometry is forced by the topological constraints to warp in response. Although this warping does not change the classical cancellation of the brane tensions in the effective 4D cosmological constant, it likely *does* ruin the argument that the quantum contributions remain as small as 10^{-12} GeV. This is because in the warped case the basic estimate for the quantum corrections remains $\delta\rho \sim m_{\text{KK}}^4$, where m_{KK} is a typical Kaluza-Klein mass for the bulk modes. Now, in the unwarped case this mass must be very small, $m_{\text{KK}} \sim v \sim 1/r$, in order to have the correct hierarchy between the strength of the electroweak and gravitational interactions (*i.e.* solution to the hierarchy problem). The KK mass need *not* be this small for the warped geometries, since for these part of the electroweak hierarchy can be understood in terms of the warping itself, *à la* Randall and Sundrum [30]. As a result, for warped geometries the size of the internal dimensions which is required to ensure the correct electroweak hierarchy implies a Kaluza-Klein scale m_{KK} , which is much *larger* than v . As such, the naive estimate $\delta\rho \sim m_{\text{KK}}^4$ is also much larger for these geometries than the presently-observed dark energy density.

This shows that if the SLED is to provide a successful explanation of the presently-small cosmological constant, it must explain why at present the extra-dimensional geometry is large enough to explain the electroweak hierarchy *and* why the internal dimensions are not warped. This is a dynamical issue whose resolution requires an understanding of the as-yet-unsolved issue of radius stabilization in these models. (See, however, ref. [31, 32] for some first steps towards understanding this issue within the SLED context.)

3.4 How Big Are Quantum Effects?

The central issue in the SLED proposal is to justify the estimate $\delta\rho \sim m_{sb}^4$ of the

contribution to ρ due to quantum effects involving loops of bulk fields. Although this is most properly done through explicit calculations using geometries such as the rugby ball, along the lines of ref. [17], there is much to be learned from explicit power-counting arguments.

As described earlier, the generic 4D estimate that integrating out a particle of mass M generates a vacuum energy density of order M^4 *does* apply to the energy density generated by integrating over the 4D brane-bound modes. This naturally leads to brane tensions which are of order M^4 , where M is a scale in the TeV region. Crucially, in SLED this is a localized energy distribution which sets up an extra-dimensional gravitational field, as opposed to being a direct contribution to the effective 4D cosmological constant. As such it is cancelled by the classical integration over the bulk modes, as argued above and in ref. [3].

At first sight the quantum integration over bulk modes is more problematic, since these can be regarded as an infinite Kaluza-Klein tower of 4D modes. Although some of these modes are comparatively light — with masses of order $m_{sb} \sim v$ — there are elements of the tower which are as much more massive, including masses of order M . One might expect the sum over all such modes to again contribute a contribution of order M^4 . Of course additional symmetries like supersymmetry can ameliorate this conclusion by enforcing cancellations between bosons and fermions, and this requires $\delta\rho$ to vanish in the limit that the bulk supersymmetry-breaking scale, m_{sb} , goes to zero. But in itself this could allow contributions of order $\delta\rho \sim M^2 m_{sb}^2$, and although these are much smaller than M^4 they are much larger than m_{sb}^4 .

A more geometrical idea of how scales like $M^2 m_{sb}^2$ might arise can be had by considering the types of local effective interactions which can be generated by integrating out a bulk mode whose mass is of order M . The key difference for SLED over other cosmological constant proposals is that 6D general covariance requires this effective interaction to be local in six-dimensions, for all M 's right down to the scale of the observed cosmological constant, v . For instance, some typical six-dimensional interactions which could be generated in this way have the schematic form

$$S_{\text{eff}} = \int d^6x \sqrt{-g_6} \left[c_0 M^6 + c_1 M^4 R + c_2 M^2 R^2 + c_3 \log(M/\mu) R^3 + \dots \right], \quad (3.2)$$

where R is the 6D curvature scalar and the arbitrary constants, c_i , are dimensionless and are taken to be $O(1)$. μ here denotes an arbitrary scale whose value is not important for the present purposes. The ellipses describe terms involving the other fields of 6D supergravity which involve the same number of derivatives, as well as other curvature invariants *etc.*.

Evaluating this action at the SLED vacuum configuration implies $R \sim 1/r^2$, and — keeping in mind the volume of the internal two dimensions is order r^2 — we see that S_{eff} generates the following ultraviolet-sensitive terms in the effective 4D scalar

potential for r :

$$\delta V_{\text{eff}} = c_0 M^6 r^2 + c_1 M^4 + c_2 M^2 / r^2 + c_3 \log(M/\mu) / r^4 + \dots \quad (3.3)$$

Clearly it is the terms in S_{eff} which are proportional to positive powers of M which are dangerous, and whose absence in any explicit quantum calculation (like those of [17]) must be explained.

It is 6D supersymmetry (and general covariance) which disposes of the terms of order M^6 and M^4 in S_{eff} . The terms of order M^6 are excluded because supersymmetry forbids a bare cosmological constant in six dimensions, and so enforces $c_0 = 0$. A unique set of terms of order M^4 are allowed by 6D supersymmetry, and this is precisely the classical supergravity action with which we start. A quantum contribution to c_1 can be regarded as a renormalization of the classical action, and so does not change the above arguments. It does not because the precise value of the classical couplings (like the 6D Newton constant) is not important for the cancellation of brane tension and bulk curvature.

The potentially dangerous terms in S_{eff} are those which involve squares of the curvature together with their partners under supersymmetry [3]. Furthermore, explicit one-loop field-theoretic calculations [33] show that the coefficients, c_2 , of these terms are not generically zero in specific 6D supersymmetric field theories. In order to see how large loop contributions to c_2 can be, it is useful to borrow the counting of coupling constants which would follow if the 6D theory is regarded as the low-energy limit of string theory.⁷ In this case it is the expectation of the 6D dilaton itself which tracks loops, and the rest of this section argues that because of the small size predicted for the dilaton field it is only the one-loop contribution to the curvature-squared contributions which need vanish in order to keep the effective 4D cosmological constant small enough.

To see how this works it is useful to make the 6D dilaton explicit in the above estimates, since it is the *v.e.v.* of the dilaton which plays the role of the bulk coupling. This is most easily performed in the string frame — for which the classical action has the form

$$S_{\text{cl}} = c_1 \int d^6 x \sqrt{-\hat{g}} e^{-2\phi} \left[\hat{R} + \dots \right] . \quad (3.4)$$

In this frame an ℓ -loop correction is accompanied by a power $e^{2(\ell-1)\phi}$ (showing that the classical contribution, eq. (3.4), corresponds to tree level: $\ell = 0$).⁸

The size of the cosmological constant, on the other hand, is most directly seen in the Einstein frame, which is defined by rescaling the metric so that the coefficient

⁷It is natural to think in terms of string theory to describe the ultraviolet behaviour of this model, since we are interested in massive states whose scale, M , is comparable with the scale of gravitational physics in the extra dimensions.

⁸It is the common appearance of $\exp(-2\phi)$ in front of the Einstein, Maxwell and brane actions which suggests looking for this 6D model as a low-energy limit of Heterotic string vacua compactified on $K3$ in the presence of NS5-branes [34].

of R is independent of ϕ . In 6 dimensions this frame is related to the string frame by the requirement $\hat{g}_{MN} = e^\phi g_{MN}$, so that $\sqrt{-\hat{g}} e^{-2\phi} \hat{R} = \sqrt{-g} R$, and so an ℓ -loop contribution to a curvature-squared term becomes of order

$$\sqrt{-\hat{g}} \hat{R}^2 = \sqrt{-g} e^{(2\ell-1)\phi} R^2. \quad (3.5)$$

This shows that the dangerous contribution to V_{eff} is of order $c_2 M_p^2 e^\phi / r^2$. To estimate the size of this contribution requires knowing the size of ϕ , which the explicit solutions to the 6D field equations show to be $e^\phi \sim 1/(Mr)^2$ [25, 26, 20]. Consequently we see that the one-loop-generated curvature-squared terms are dangerous — being of order $M_p^2/(M^2 r^4) = M^2/r^2$, where the relation $M_p = M^2 r$ is used. By contrast, the contributions at two loops and beyond are smaller than $O(m_{sb}^4)$ because of the extremely small size of the bulk coupling, e^ϕ . Within the SLED proposal the only dangerous bulk-loop contributions to the cosmological constant arise at one loop, making their vanishing easier to arrange than usual. Work is underway to determine the general conditions which the vanishing of this one-loop contribution requires.

One might worry that there may be phenomenological difficulties with having bulk couplings this small, but this is not a difficulty for particles localized on the brane since their couplings are not proportional to e^ϕ . (That is, because of the condition $\lambda = 0$ in eq. (3.1), which was required by the condition that the brane couplings not break the classical bulk scale invariance.) This shows that it is precisely the dilaton/brane coupling which ensures the classical cancellation between brane tensions and bulk curvatures [19] which also ensures that brane couplings are not suppressed by the small size of the bulk coupling, e^ϕ .

3.5 Consistency With Cosmology and Observations?

Some of the cosmological implications of the 5 dimensional ‘self-tuning’ models have been explored [35], inasmuch as they are believed to change the form of the effective Friedmann equation as seen by observers during or after Big Bang Nucleosynthesis [36]. One might wonder whether models of the SLED type might be ruled out on cosmological grounds even if one were to suspend judgement on microscopic naturalness issues.⁹

There are two aspects to consider here. One might first ask whether the 6D models predict deviations from the effective 4D Friedmann equations in the same way as can happen for 5D models. Although the general statement in 6 or higher dimensions is not known, we do know that these corrections in 5D are suppressed if the energies of brane fields are small compared with the brane tensions, $T \sim M^4$. As

⁹As mentioned earlier, we do not consider here potential problems at epochs before nucleosynthesis, such as the LED and SLED problems of refs. [8, 13, 6], since these are not robust to changes to the details of the early universe cosmology or of the particle physics of the model.

such change can be expected to be less and less important at lower and lower energies, and so are not likely to be the most dangerous worries at and after nucleosynthesis, for which typical energy densities are much smaller than M^4 .

A more pressing concern at late cosmological times is that the small nonzero vacuum energy is likely to arise in a radius-dependent way, and so the vacuum energy will likely really be a radion potential, $\rho = V(r) \sim v^4 \sim 1/r^4$. Since r is dynamical, and given that V is minimized for large $r \sim 1/v$, it is energetically possible for r to be rolling cosmologically during the present epoch. Since this potentially leads to a quintessence-like picture [37] for the dark energy, it is subject to two very strong observational constraints.

The first bound arises in such a picture because the fluctuations of r are described by an extremely light scalar, whose mass is of order $m_r \sim (M_p r^2)^{-1} \sim 10^{-32}$ eV, and which should couple to matter with gravitational strength.¹⁰ If so, why doesn't this kind of model fall afoul of the strong observational constraints on modifications to gravity at large distances [38]?

The second bound comes because the value of r determines the relative strength of the gravitational and other forces, and so a conflict with observations can arise if r rolls appreciably during the cosmological evolution after Big Bang Nucleosynthesis. This point has been forcefully raised within the context of the SLED proposal by ref. [40].

Both of these constraints were explicitly investigated for a closely related kind of quintessence model in ref. [39]. There it was found to be possible (but not automatic) to satisfy both constraints and so to construct realistic cosmologies. It was found in this reference that these bounds can cause trouble for generic evolution of the scalar r , but that phenomenologically acceptable cosmologies were also allowed for a reasonably wide range of initial configurations. In essence the constraints on changes to M_p over cosmological time scales tends to be evaded because the r motion is heavily over-damped by cosmic friction. The constraints on long range forces can be satisfied because the matter couplings of the light scalar are r -dependent, and so can evolve towards small couplings at the present epoch (which is the only time where such bounds apply).

Of course, this should only be regarded as an existence proof that these observationally-successful cosmologies are possible, and a more detailed studies can and should be done given an explicit prediction for $V(r)$.

4. Conclusions and Open Issues

It is worth closing with a brief summary of the advantages of, and challenges remain-

¹⁰Interestingly, such a small mass for a gravitationally-coupled scalar can only be stable under renormalizations in 4 dimensions if the world becomes extra-dimensional at scales $\Lambda \sim (M_p m_r)^{1/2} \sim v$ [39].

ing for, the SLED proposal. On the one hand, it provides the following uniquely attractive new perspectives on the cosmological constant problem:

- By far the strongest motivation for the SLED picture is the understanding it provides as to why the well-understood particles of ordinary experience (like the electron) do not contribute unacceptably to the cosmological constant. There are two conceptual points which allow it to do so. The first is the observation that the world is six-dimensional at the energies of ordinary interest, which implies that the zero-point energy of particles like the electron are localized sources of curvature in the extra dimensions rather than directly a contribution to the cosmological constant. It is the extra-dimensional curvature which this localized energy produces which cancels the potentially large contributions of the brane tensions to the effective 4D cosmological constant.
- Once the brane fields are integrated out, the same must then also be done for the bulk modes. This leads to the second important conceptual point of the SLED proposal, which is the observation that the bulk gravitational sector is much more supersymmetric than is the sector involving ordinary matter. In the SLED picture ordinary matter is *not* approximately supersymmetric, and at the weak scale need not resemble the supersymmetric standard model (minimal or otherwise). Nevertheless, this supersymmetry breaking on the brane only causes a small amount of supersymmetry breaking in the gravitational sector, $m_{sb} \sim 10^{-3}$ eV, because of the weak strength of the gravitational couplings. It is this small supersymmetry breaking in the bulk which explains why the 4D cosmological constant is nonzero but small, rather than being strictly zero.
- Although it is motivated by the cosmological constant problem, the SLED proposal has rich experimental implications for high-energy accelerators. Since gravitational physics must be at the TeV scale, there should be observable signals of extra-dimensional gravity at TeV-scale accelerators like the Large Hadron Collider just as for the non-supersymmetric LED picture.

This would look very different from what would be expected for supersymmetric extensions of the Standard Model. In its minimal form — what might be called ‘mSLED’ — the description of physics between the cosmological constant scale (10^{-3} eV) and the 6D gravity scale (10 TeV) is given by a relatively simple action $S = S_b + S_B + S_{int} + S_{ob}$. The first two of these terms describe the physics on our brane and the physics of the bulk, and are very precisely known. The minimal choice for the physics on our brane, S_b , is the Standard Model itself. The minimal choice for the physics of the bulk is described by the 6D Nishino-Sezgin action, S_B , (or its ungauged limit). S_{int} describes the interactions between the bulk and our brane, and is less well understood. It

need not be supersymmetric, and should be taken to be the most general possible form consistent with all unbroken symmetries. S_{ob} describes the unknown physics residing on any other branes which may be situated about the compact 2 dimensions.

- There are also likely to be many experimental implications for non-accelerator physics. These include deviations from the Newtonian inverse-square law at sub-millimeter scales, scalar-tensor-type deviations from gravity over long-distances; and potential implications for astrophysics, along the lines of the bounds of ref. [8, 12, 6].
- Finally, to the extent that the energy cost for changing r is as small as v^4 , this field is very likely to be cosmologically rolling today, leading to a quintessence-type cosmology for the dark energy, possibly along the lines of ref. [39]. In this case there is the attractive possibility of relating cosmological observations of the dark energy to the evolution of the shape of the extra dimensions. The effective 4D field theory would in this case consist of the low-energy Standard-Model limit coupled to any very-low energy particles on other branes and to the massless modes of the bulk.

Much remains to be done however, including the following open issues (on which work is currently underway).

- Above all, the main result which remains to be established is the verification of the size of the 4D cosmological constant produced by an explicit quantum integration over the bulk modes. One way to do so is by finding an explicit derivation of the 6D Nishino-Sezgin model as a vacuum within string theory, since this allows an accurate identification of which fields count string loops.¹¹
- It would be instructive to have explicit one-loop calculations to verify how the renormalization of the topological constraints is not changed as one integrates out scales between M and v .
- The dynamics of the bulk geometry and its implications for cosmology are in principle calculable for various 6D vacua (such as spheres or torii). An understanding of this dynamics is crucial to understanding why the bulk radius should stabilize at such large values, $r \sim 1/v$, and why the bulk geometry should remain unwarped (as it must if the vacuum energy is to be as small as is observed).
- The detailed phenomenological implications of the SLED proposal need to be worked out in detail, for both accelerator and non-accelerator applications.

¹¹See ref. [41] for recent progress along these lines.

If these ideas are right, the next decades may bring a fruitful interplay between microscopic and cosmological observations. We should be so lucky!

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